ACTIVE CONTROL OF A QUARTER-CAR MODEL SUSPENSION SYSTEM WITH SLIDING MODE AND PID CONTROL APPROACHES

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Abstract

This study covers a State Dependent Ricatti Equation (SDRE) based Sliding Mode Controller (SMC) and classical PID controller that are applied for an active control of the suspension system of a quarter car model. At first, this suspension system is controlled by SDRE based sliding mode controller for a road disturbance input and then for the same input the same system is controlled by classical PID controller. Finally, the results are compared.

Key Words: Suspension systems, SDRE Based Sliding Mode Control, PID controller

1. Introduction

Conventional vehicle suspension systems are used for isolation of road-induced vibration. There are three common types of suspension systems. These are active, semiactive and passive suspensions. Passive suspension system means that it has springs, dampers or shock absorbers. On the other hand, the semi active suspension systems have controllable dampers as electrorheological (ER) and magnetorheological (MR). Both fluids are smart materials made mixing fine particles into a liquid with low viscosity. Active suspensions mean that at least some part of required suspension force is generated from active power sources such as compressors, hydraulic pumps, etc. In application of active suspensions, microprocessors, associated electronic devices and also actuators are used. Compared with passive suspensions, active suspensions can improve the performance of the suspension system over a wide range of frequency[1-3].

The present work includes active control of an quarter car model subjected to random road disturbance input. For control of active power source, both SDRE based sliding mode and classical PID controllers are used. The main purpose of this study is to consume less energy by generating less force while the road-induced vibrations are isolating.

The mathematical backround of controllers are given in Section 2 of the paper. In Section 3, mathematical model of car and suspension system are given. The simulation results are given in Section 4 and finally conclusions are in Section 5.

2. Materails and Method

2.1 Theory

In this study, two types of control techniques are used. The first technique is SDRE based SMC and the second technique is classical PID controller.

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The system to be controlled is describes as follows.

$$\dot{x}(t) = A(x)x(t) + Bu(t) \tag{2.1}$$

Where $x \in R^n$ and $u \in R^m$ are state and control vectors. Since the system is considered to be frozen at each time intervals, the system is approached as a LTI system at that time intervals. Therefore, the system becomes as follows.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{2.2}$$

Then if the coordination transformation is used on that system as follows.

$$z(t) = T_r x(t) \tag{2.3}$$
$$T_r = q(B) \tag{2.4}$$

Where $q(B)$ is Q-R decomposition matrix of matrix B. And then the system and control matrix in terms of that decomposition can be written as follows.

$$A_{reg} = T_r A T_r{}^T \tag{2.5}$$
$$B_{reg} = T_r B \tag{2.6}$$

For SDRE algorithm, a performance index must be determined. That cost funciton is given as

$$J = \frac{1}{2}\int_{t_s}^{\infty} x(t)^T Q x(t) dt \tag{2.7}$$

where $Q \in R^{nxn}$, symmetric and positive definite matrix. In addition, t_s is the time when the system begins to sliding motion. At that time $x(t) \rightarrow 0$. After that if the coordinate transformation above is applied on the matrix Q.

$$Q_{reg} = T_r Q T_r{}^T \tag{2.8}$$

Matrices in Equation (2.4), Equation (2.5) and Equation (2.8) are defined as in [4]

$$A_{reg} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{2.9}$$

$$B_{reg} = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \tag{2.10}$$

Where $A_{11} \in R^{(n-n-1)x(n-m-1)}$, $A_{12} \in R^{(n-m-1)x(m-1)}$, $A_{21} \in R^{(m-1)x(n-m-1)}$, $A_{22} \in R^{(m-1)x(m-1)}$ and $B_2 \in R^{(m-1)x(m-1)}$.

$$T_r Q T_r^T = \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix} \tag{2.11}$$

Then if the variables in Equation (2.3) and Equation (2.11) is introduced into Equation (2.7), the performance index is defined as

$$J = \frac{1}{2}\int_{t_s}^{\infty} z_1^T Q_{11} z_1 + 2z_1^T Q_{12} z_2 + z_2^T Q_{22} z_2 \, dt \tag{2.12}$$

But performance index in Equation (2.12) is not in standard LQR form. Because of this the variable $2z_1^T Q_{12} z_2$ in Equation (2.12) must be eliminated. For elimination the steps given below should be followed.

$$2z_1^T Q_{12} z_2 + z_2^T Q_{22} z_2 = \left(z_2 + Q_{22}^{-1} Q_{21} z_1\right)^T \cdot Q_{22}\left(z_2 + Q_{22}^{-1} Q_{21} z_1\right) - z_1^T Q_{21}^T Q_{22}^{-1} Q_{21} z_1 \tag{2.13}$$

Then, if new variables are defined as follows.

$$\tilde{Q} = Q_{11} - Q_{12} Q_{22}^{-1} Q_{21} \tag{2.14}$$

And

$$v = z_2 + Q_{22}^{-1} Q_{21} z_1 \tag{2.15}$$

The new cost funtion is obtained after the performance index in Equation (2.12) is redefined with the new variables in Equation (2.13), Equation (2.14) and Equation (2.15) as

$$J = \frac{1}{2}\int_{t_s}^{\infty} (z_1{}^T \tilde{Q} z_1 + v^T Q_{22} v) dt \tag{2.16}$$

Therefore, performance index in Equation (2.16) have transformed into its regular form. Then if we rewrite coordinate transformation as

$$\dot{z}_1(t) = A_{11} z_1(t) + A_{12} z_2(t) \tag{2.17}$$

And if a new parameter is defined as below

$$\widehat{A} = A_{11} - A_{12} Q_{22}^{-1} Q_{21} \tag{2.18}$$

Then if the parameters in Equation (2.15) and Equation (2.18) are inserted into Equation (2.17) the result is as folowing.

$$\dot{z}_1(t) = \widehat{A}z_1(t) + A_{12}v(t) \tag{2.19}$$

If the performance index in equation (2.16) is solved by using state dependent Riccati equation the solution is obtained as

$$P_1\widehat{A} + \widehat{A}^T P_1 - P_1 A_{12} Q_{22}^{-1} A_{12}^T P_1 + \breve{Q} = 0 \tag{2.20}$$

In equation (2.20) P_1 is positive definite and solution of the Riccati equation. Then if an optimal v is defined as

$$v = -Q_{22}^{-1} A_{12}^T P_1 z_1 \tag{2.21}$$

Then if this optimal term is equalized with the term in equation (2.15) and after that if new equation is solved the result is given below.

$$z_2 = -Q_{22}^{-1}(A_{12}^T P_1 + Q_{21})z_1 \tag{2.22}$$

Finally from the equaiton (2.22), the sliding surface's slope is obtained as follows.

$$M = -Q_{22}^{-1}(A_{12}^T P_1 + Q_{21}) \tag{2.23}$$

In equation (2.23) $M \in R^{n \times m}$. After finding the slope, the sliding surface is obtained as

$$\sigma(z) = z_2 + Mz_1 \tag{2.24}$$

To produce further, a control term which hold the system on designed sliding surface should be defined. For this reason, the condition below must be satisfied [4].

$$\text{For } \sigma(z) > 0 \ \dot{\sigma}(z) < 0, \text{ for } \sigma(z) < 0 \ \dot{\sigma}(z) > 0 \tag{2.25}$$

Here, $\dot{\sigma}(z)$ is the first time derivative of the sliding surface.

$$\dot{\sigma}(z) = MAz \tag{2.26}$$

The control term is divided into two parts as follows.

$$u = u_{eq} + u_{non} \tag{2.27}$$

$$u_{eq} = -(\sigma B)^{-1}\dot{\sigma} \tag{2.28}$$

$$u_{non} = -k(t,z)(\sigma B)^{-1} sgn(\sigma) \tag{2.29}$$

Here, $k(t,z) > 0$. Before advancing to the next step, it is first pertinent to establish sufficient conditions which guarantee that an ideal sliding motion will take place. Intuitively, the sliding surface must be at least locally attractive. This may be expressed mathematically as

$$\lim_{\sigma \to 0^+} \dot{\sigma} < 0 \ \text{ and } \ \lim_{\sigma \to 0^-} \dot{\sigma} > 0 \tag{2.30}$$

In some domain $\Omega \subset R^n$. In this case the sliding surface would be

$$D = \sigma \cap \Omega = \{x \in \Omega \ : \ \sigma(x) = 0\} \tag{2.31}$$

The expression given in equation (2.30) is often replaced by the equivalent, but more succinct criterion [4]

$$\sigma\dot{\sigma} < 0 \tag{2.32}$$

The expressions in equation (2.30) and (2.32) are termed reachability conditions. In general, if the reachability condition in equation (2.32) is satisfied globally then since

$$\frac{1}{2}\frac{d}{dt}\sigma^2 = \sigma\dot{\sigma} \tag{2.33}$$

It follows that the function

$$V(\sigma) = \frac{1}{2}\sigma^2 \tag{2.34}$$

is a Lyapunov function for the state σ. Unfortunately, although equations (2.30) and (2.32) are commonly encountered in the literature, they do not guarantee the existence of an ideal sliding motion [4]. Essentially these conditions only guarantee that the sliding surface is reached asymtotically.

A stronger condition, guaranteeing an ideal sliding motion, is the η-reachability condition given by [4]

$$\dot{\sigma}\sigma \le -\eta|\sigma| \tag{2.35}$$

Here, η is a small positive constant. By rewriting equation (2.33) a

$$\frac{1}{2}\frac{d}{dt}\sigma^2 \leq -\eta|\sigma| \tag{2.36}$$

and integrating from 0 to t_s, it follows that

$$|\sigma(t_s)| - |\sigma(0)| \leq -\eta t_s \tag{2.37}$$

and thus the time taken to reach $\sigma = 0$, represented by t_s, satisfies condition below[4,5].

$$t_s \leq \frac{|\sigma(0)|}{\eta} \tag{2.38}$$

and for PID control

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{2.39}$$

where K_p, K_i, K_d and $e(t)$ are proportional, integral, derivative gain constants and error function respectively[6].

3. Dynamics of Car Model

In this study, a simple quarter-car suspension model that consists of one-fourth of the body mass, suspension components and one wheel is shown in Figure 1. This model has been used extensively in the literature and captures many essential characteristics of a real suspension system. The equations of motion and figure are borrowed from [7].

$$m_s\ddot{z}_s + c_s[\dot{z}_s(t) - \dot{z}_u(t)] + k_s[z_s(t) - z_u(t)] = -u(t) \tag{3.1}$$
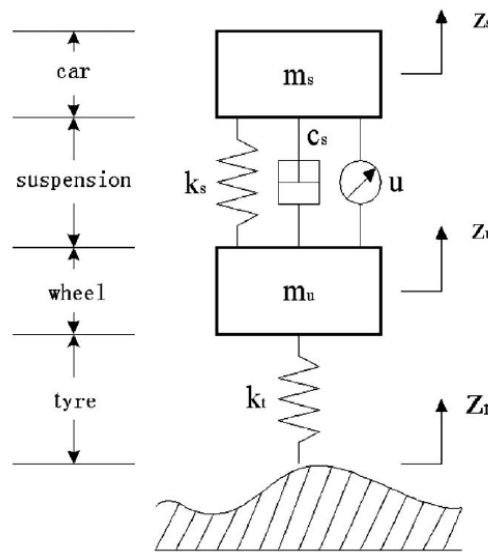$$m_u\ddot{z}_u + c_s[\dot{z}_u(t) - \dot{z}_s(t)] + k_s[z_u(t) - z_s(t)] + k_t[z_u(t) - z_r(t)] = u(t) \tag{3.2}$$

Figure 1. Quarter-car model [7]

where m_s is the sprung mass, which represents the car chassis; m_u is the unsprung mass, which represents the wheel assembly; c_s and k_s are damping and stiffness of the uncontrolled suspension system, respectively; k_t serves to model the compressibility of the pneumatic tyre; z_s and z_u are the displacements of the sprung and unsprung masses, respectively; z_r is the road displacement input; $u(t)$ represents the external input force of the suspension system.

For sliding mode control Equation (3.1) and (3.2) must be transform to form below,

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{3.3}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -k_s/m_s & 0 & -c_s/m_s & c_s/m_s \\ k_s/m_u & -k_t/m_s & c_s/m_s & -c_s/m_s \end{bmatrix} \tag{3.4}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \dfrac{-1}{m_s} \\ \dfrac{1}{m_u} \end{bmatrix} \tag{3.5}$$

$$B_w = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \tag{3.6}$$

After choosing the states as

$$x_1(t) = z_s(t) - z_u(t) \tag{3.7}$$
$$x_2(t) = z_u(t) - z_r(t) \tag{3.8}$$
$$x_3(t) = \dot{z}_s(t) \tag{3.9}$$
$$x_4(t) = \dot{z}_u(t) \tag{3.10}$$
$$x(t) = [x_1(t)\, x_2(t)\, x_3(t)\, x_4(t)], w(t) = \dot{z}_r(t) \tag{3.11}$$

where $x_1(t)$ represents suspension deflection, $x_2(t)$ is the tyre deflection, $x_3(t)$ is the sprung mass speed, $x_4(t)$ is unsprung mass speed and $w(t)$ denotes the disturbance caused by road roughness.

And for PID control, the analyzes must be done in s domain. Therefore, the transfer functions between inputs and outputs must be determined. In that analyze x_s is used as output and $u(t)$ and $d(t)$ are used as input. So for defining the first transfer function between x_s and $d(t)$ the laplace transform of Eq(3.1) and (3.2) must be taken as follows,

$$0 = m_s s^2 X_s + k_s X_s - k_s X_u + c_s s X_s - c_s s X_u \tag{3.12}$$

From Eq(8)

$$X_u = X_s \left(\frac{m_s s^2 + c_s s + k_s}{c_s s + k_s} \right) \tag{3.13}$$

Then if the laplace transform of Eq(3.2) is taken and rearrange with Eq(3.13), the first transfer function between x_c and w wil be like that,

$$Tf1 = \frac{c_s k_t s + k_s k_t}{\Delta} \tag{3.14}$$

Where

$$\Delta = m_s m_u s^4 + s^3(m_s c_s + m_u c_s) + s^2(m_s k_t + m_s k_1 + m_u k_1) + c_s k_t s + k_s k_t \tag{3.15}$$

And if the same calculations is done the second transfer function between x_s and $u(t)$ will be as follows,

$$Tf2 = \frac{-m_u s^2 - k_t}{\Delta} \tag{3.16}$$

4. Simulation Results

In this section, firstly, SDRE based sliding mode controller applied to the quarter car model. The models parameters have following values [7].

Table 1. Parameters of quarter car model

Parameters	Values
m_s	504.5 kg
m_u	62 kg
k_s	13100 N/m
c_s	400 Ns/m
k_t	252000 N/m

The random input of disturbance is shown in Figure 2; the responses of suspension deflection, tyre deflection, sprung mass acceleration and force input are shown in Figure 3-6 respectively.
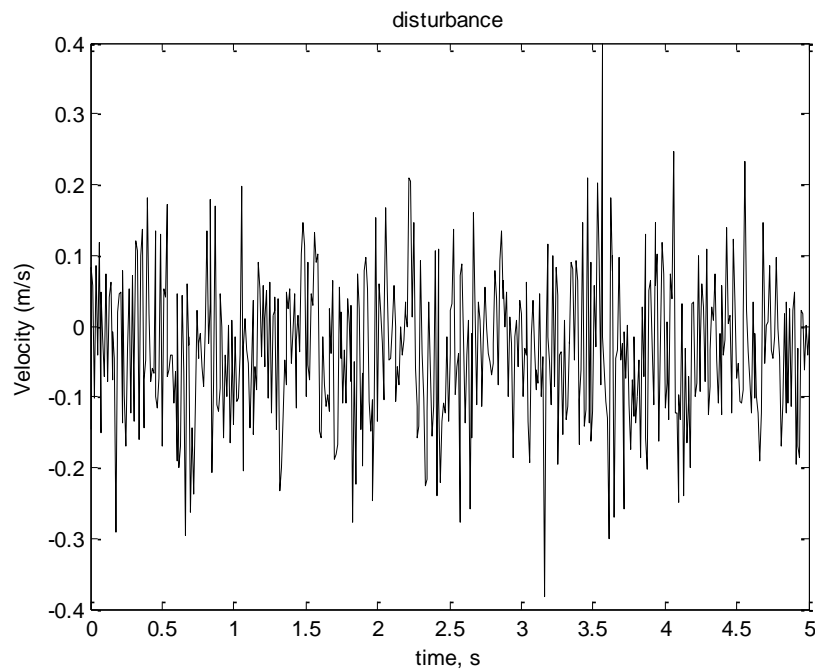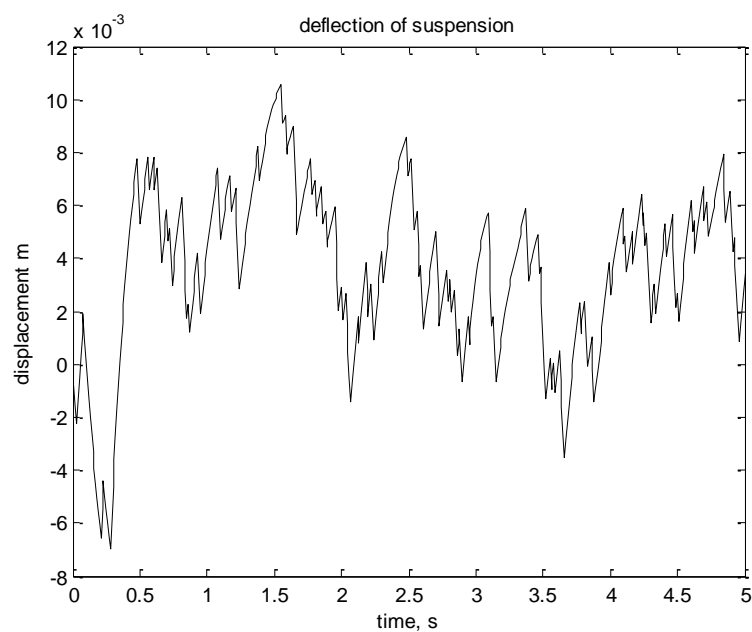

Figure 2. Disturbance Signal
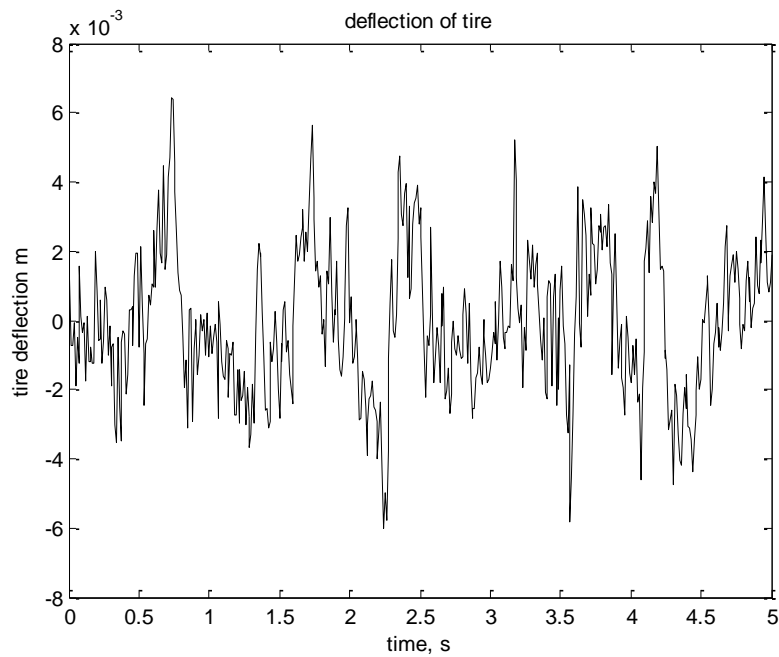

Figure 3. Deflection of Suspension
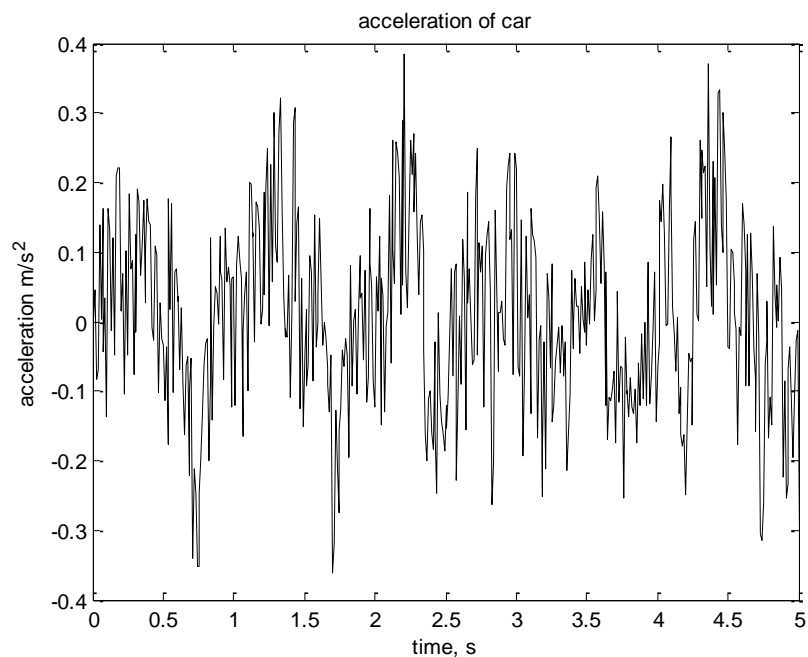
Figure 4. Deflection of tire

Figure 5. Acceleration of car

In Figure 2 the random input is choosen randomly. For applications deflection of tire and deflection of suspension must be small values. According to the Figure 3 and Figure 4, it can be seen that the values are small in comparison to [7]. In Figure 5 the result of vertical acceleration caused by random input is shown. This value should also be small. If this value is not small displacement of car value will also not be small and that means big oscillations in vertical direction.
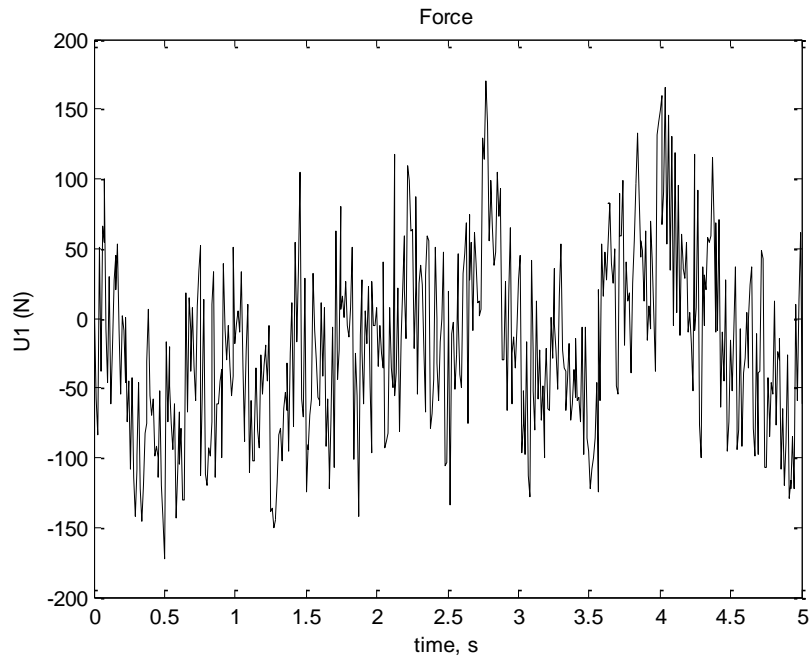
Figure 6. Force Input

In Figure 6 the desired input force input result is seen. This force realised by a force generator. The k value of SDRE based SMC is choosen 1.0 after some trials. In comparison to [7] this value is small and it means that the system is comsumed less energy.

For now, PID control technique is applied. For PID control is chosen so control system scheme is as follows,
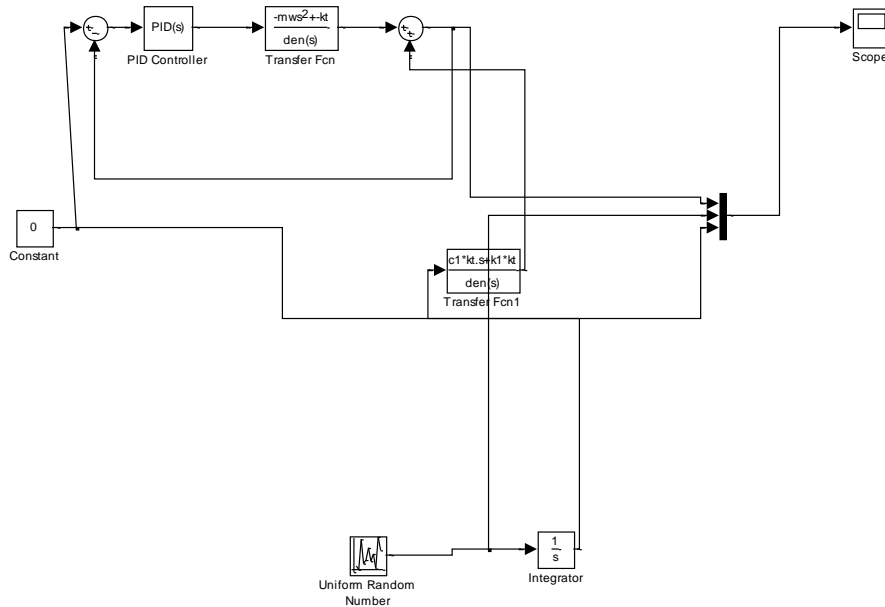

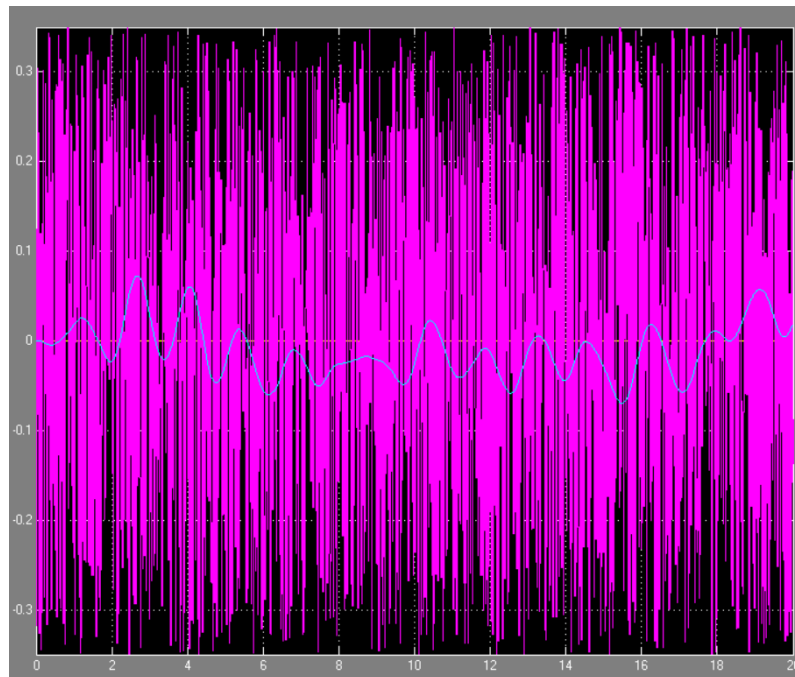Figure 7. PID Control scheme of model

Figure 8. Displacement of car body and Disturbance Input

In Figure 8, the purple graph is road disturbance model and yellow line is the desired output for that disturbance and blue line is the displacement of car body. The PID constants are choosen 0.1, 0.5 and 5 after some trials.

5. Conclusion

In our study, an active vibration control of suspension system for quarter car body with two different controller. The first one is SDRE based sliding mode controller. In that controller, the sliding surfaces are designed by SDRE method. Then, these slopes are used for designing of sliding surfaces. The same model is also controlled by [7]. They use use both active, passive and semi active vibration control. But mainly they use MR damper for suspension system with H_∞ control algorithm. In our study, the suspension system is give better results with consuming less energy. The k value in SDRE based SMC algorithm can be changed. If a better value for k is found the results will be better and more acceptable.

Lastly, the same system is controlled by classical PID. As seen in Figure 8. the desired value is not tracked accurately but acceptable.

References

[1] Hrovat D. Survey of advanced suspension developments and related optimal control applications. Automatica 1997,Vol: 33 No:10,1817-1781.
[2] Yao G. Z.,Yap F. F., Chen G., Li W.H, Yeo S.H., MR damper and its application for semi active control of vehicle suspension system, Mechatronics 2002: 12: 973–963.
[3] Verros G., Natsiavas S., Papadimitriou C. Design optimization of quarter-car models with passive and semi-active suspensions under random road excitation. Journal of Vibration and Control 2005:11:606-581.
[4] Edwards, C. and Spurgeon, S.K. Sliding Mode Control, Theory and Applications. London: Taylor&Francis; 1998, 1-75.

[5] Bicer O., Salamcı M.U. A new sliding mode controller design for satellite position control under effect of unmatched disturbances. Otomatik Kontrol Ulusal Toplantısı TOK' 2015 Denizli, 279-274.

[6] Ogata K. Modern Control Engineering. 4rd ed. New Jersey:Prentice Hall; 2002.

[7] Haiping Du, Kam Yin Sze, James Lam Semi-active H_∞ control of vehicle suspension with magneto-rheological dampers, Journal of Sound and Vibration 2005:283: 996–981.